Black Hole Perturbation Theory: An Introduction

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July 22, 2024

Overview

1. Previously
2. The Master Equations
    - Regge-Wheeler
    - Zerilli

Previously

Recall

→ The spherical tensors and its orthogonality relations [2]:

- Scalar → Y^{lm}:

$$\langle Y^{lm}, Y^{l'm'} \rangle = \delta^{ll'} \delta^{mm'} \tag{1}$$

- Vector → Y_a^{lm} (even), S_a^{lm} (polar):

$$\begin{aligned} \gamma^{ab} \langle S_a^{lm}, S_b^{l'm'} \rangle &= l(l+1) \delta^{ll'} \delta^{mm'} \\ \gamma^{ab} \langle Y_a^{lm}, Y_b^{l'm'} \rangle &= l(l+1) \delta^{ll'} \delta^{mm'} \\ \gamma^{ab} \langle Y_a^{lm}, S_b^{l'm'} \rangle &= 0 \end{aligned} \tag{2}$$

- Tensor → Z_{ab}^{lm} (even), S_{ab}^{lm} (polar):

$$\begin{aligned} \gamma^{ac} \gamma^{bd} \langle S_{ac}^{lm}, S_{bd}^{l'm'} \rangle &= l(l+1)(l+2) \delta^{ll'} \delta^{mm'} \\ \gamma^{ac} \gamma^{bd} \langle Z_{ac}^{lm}, Z_{bd}^{l'm'} \rangle &= l(l+1)(l+2) \delta^{ll'} \delta^{mm'} \\ \gamma^{ac} \gamma^{bd} \langle Z_{ac}^{lm}, S_{bd}^{l'm'} \rangle &= 0 \end{aligned} \tag{3}$$

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→ The spherical tensors and its orthogonality relations (for fixed l, m):

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- Vector → $e_a^{e,lm}$, $e_a^{o,lm}$:

$$
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$$h_{Aa} = h_A^{e,lm} e_a^{e,lm} + h_A^{o,lm} e_a^{o,lm} \tag{8}$$

$$h_{ab} = r^2 \left(K^{lm} \gamma_{ab} Y^{lm} + G^{lm} e_{ab}^{e,lm} + 2h^{lm} e_{ab}^{o,lm} \right) \tag{9}$$

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→ Parity decomposition:

$$h_{\mu\nu}(t, r, \theta, \phi) = h_{\mu\nu}^{e}(t, r, \theta, \phi) + h_{\mu\nu}^{o}(t, r, \theta, \phi), \tag{10}$$

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$$h^{o}_{\mu\nu} = \left(h^{o,lm}_{A} e^{o,lm}_{a} + 2h^{lm} e^{o,lm}_{ab} \right)_{\mu\nu} \tag{11}$$

The Regge Wheeler Gauge

→ In the Regge-Wheeler gauge:

$$h_{\mu\nu}^{odd} = \begin{pmatrix} h_0^{o,lm} \\ h_1^{o,lm} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & e_\theta^{o,lm} & e_\phi^{o,lm} \end{pmatrix} = \begin{pmatrix} 0 & 0 & h_0^{o,lm} e_\theta^{o,lm} & h_0^{o,lm} e_\phi^{o,lm} \\ 0 & 0 & h_1^{o,lm} e_\theta^{o,lm} & h_1^{o,lm} e_\phi^{o,lm} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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→ Symmetrizing:

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & h_0^{o,lm} e_\theta^{o,lm} & h_0^{o,lm} e_\phi^{o,lm} \\ 0 & 0 & h_1^{o,lm} e_\theta^{o,lm} & h_1^{o,lm} e_\phi^{o,lm} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

The Regge-Wheeler Gauge

→ Also:

$$h_{\mu\nu}^{e,lm} = r^2 K^{lm} Y^{lm} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & sin^2\theta \end{pmatrix} + Y^{lm} \begin{pmatrix} f H_0^{lm} & H_1^{lm} & 0 & 0 \\ H_1^{lm} & f^{-1} H_2^{lm} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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→ s.t.:

$$h^{e,lm}_{\mu\nu} = \begin{pmatrix} fH^{lm}_0 & H^{lm}_1 & 0 & 0 \\ H^{lm}_1 & f^{-1}H^{lm}_2 & 0 & 0 \\ 0 & 0 & r^2 K^{lm} & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta K^{lm} \end{pmatrix} Y^{lm} \tag{12}$$

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$$2\delta G_{Aa} = \alpha^{lm}_{A} e^{e,lm}_{a} + \beta^{lm}_{A} e^{o,lm}_{a} \quad (14)$$

$$2\delta G_{ab} = A^{(3)}_{lm} r^2 \gamma_{ab} Y^{lm} + s_{lm} e^{e,lm}_{ab} + t_{lm} e^{o,lm}_{ab} \quad (15)$$

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→ where:

$$\beta_0^{lm} = f(h''_{0,lm} + i\omega h'_{1,lm}) - 2i\omega\frac{f}{r}h_1^{lm} + \left(\frac{f''}{2} + \frac{l(l+1)+f-1}{r^2}\right)h_0^{lm}$$

$$\beta_1^{lm} = f^{-1}(i\omega h'_{0,lm} - \omega^2 h_1^{lm}) - 2i\omega\frac{f^{-1}}{r}h_0^{lm} + \left(\frac{f''}{2} + \frac{(l(l+1)-f-1)}{r^2}\right)h_1^{lm}$$

$$t_{lm} = i\omega f^{-1}h_0^{lm} + fh'_{1,lm} + f'h_1^{lm}$$

The Master Equations

The Regge-Wheeler Master Equation

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$$i\omega f^{-1} h_0^{lm} + f h'_{1,lm} + f' h_1^{lm} = 16\pi \delta T_s^{odd}$$

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→ Making [1]:

$$16\pi\delta T_1^{odd} = A$$

$$16\pi\delta T_s^{odd} = B$$

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$$h_0^{lm} = \frac{if}{\omega}\left(f h'_{1,lm} + f' h_1^{lm} - B\right) \tag{18}$$

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→ Taking the derivative on both sides:

$$h'_{0,lm} = \left(\frac{if}{\omega}\left(f h'_{1,lm} + f' h_1^{lm} - B\right)\right)' \tag{19}$$

The Regge-Wheeler Master Equation

→ Subs eq.:

$$h'_{0,lm} = \left(\frac{if}{\omega}\left(fh'_{1,lm} + f'h_1^{lm} - B\right)\right)' \tag{20}$$

→ into:

$$f^{-1}(i\omega h'_{0,lm} - \omega^2 h_1^{lm}) - 2i\omega\frac{f^{-1}}{r}h_0^{lm} + \frac{(l-1)(l+2)}{r^2}h_1^{lm} = A \tag{21}$$

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→ We get:

$$\left(\omega^2 + (f^2)' + \frac{1}{2}ff'' - \frac{2ff'}{r} - f\frac{(l(l+1) - f - 1)}{r^2}\right)h_1^{lm} + \\ + h''_{1,lm}f^2 + \left(3f' - \frac{2f}{r}\right)fh'_{1,lm} = -A + (fB)' - \frac{2fB}{r} \tag{22}$$

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→ Subs. into the previous equation:

$$rfQ''_{lm} + rf'Q'_{lm} + \left(\frac{r\omega^2}{f} - \frac{r}{2} f'' + f' - \frac{(l(l+1) + f - 1)}{r} \right) Q_{lm} = F_{lm}(r)$$

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→ Introducing the tortoise coordinate $r_*(r)$ where:

$$dr_* = \frac{dr}{f} \tag{24}$$

→ In these coordinates, and for $f = 1 - \frac{\sigma}{r}$:

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$$\frac{\partial^2 Q_{lm}}{\partial r_*^2} + f \frac{\left(\frac{2\omega^2 r^2}{f} - r^2 f'' + 2rf' - 2l(l+1) - 2f + 2 \right)}{2r^2} Q_{lm} = F_{lm}(r) \tag{25}$$

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$$\frac{\partial^2 Q_{lm}}{\partial r_*^2} + \left(\omega^2 - V_{odd}\right) Q_{lm} = F_{lm}(r) \qquad (26)$$

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$$\frac{\partial^2 Q_{lm}}{\partial r_*^2} + \left(\omega^2 - V_{odd}\right) Q_{lm} = F_{lm}(r) \tag{26}$$

→ where:

$$V_{odd} = \left(\frac{f''}{2} - \frac{f'}{r} - \frac{(l(l+1) + f - 1)}{r^2}\right)$$

$$F_{lm} = -16\pi \left[\delta T_1^{odd} - \delta T_s^{odd}\left(f' - \frac{2f}{r}\right) - f\left(\delta T_s^{odd}\right)'\right] \tag{27}$$

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→ Eq. 26 is known as the **Regge-Wheeler equation**.

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→ As seen in the SageMath notebook:

$$\frac{\partial^2 Z_{lm}}{\partial r_*^2} + \left(\omega^2 - V_{even}\right) Z_{lm} = S_{lm}(r) \tag{28}$$

→ which is the **Zerilli equation**

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$$\frac{\partial^2 Q_{lm}}{\partial r_*^2} + \left(\omega^2 - V_{odd}\right) Q_{lm} = F_{lm}(r)$$

$$\frac{\partial^2 Z_{lm}}{\partial r_*^2} + \left(\omega^2 - V_{even}\right) Z_{lm} = S_{lm}(r)$$

Namarië

That's All Folks!

Thank you!

Namarië

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