Cosmology: Early universe

Lesson 1

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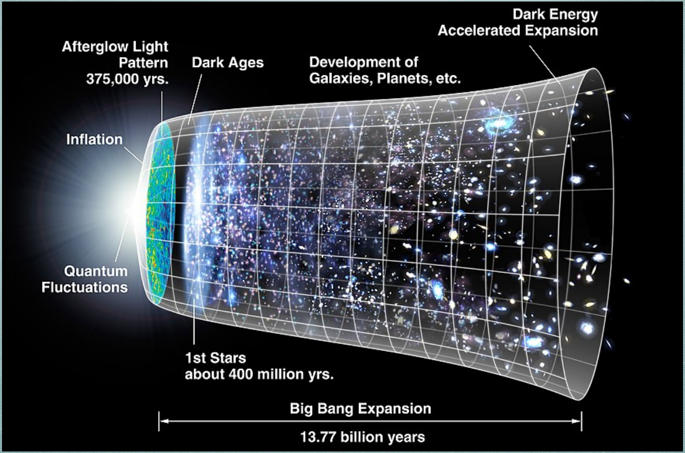
Outline

What is cosmology?

It is the branch of physics that studies the universe on a large scale: space-time and matter. This includes understanding how structure originated and its evolution into what we now know as stars, planets, galaxies, black holes, etc.

What do we mean by the early universe?

The period after the Planck era and before what is known as Big Bang Nucleosynthesis.

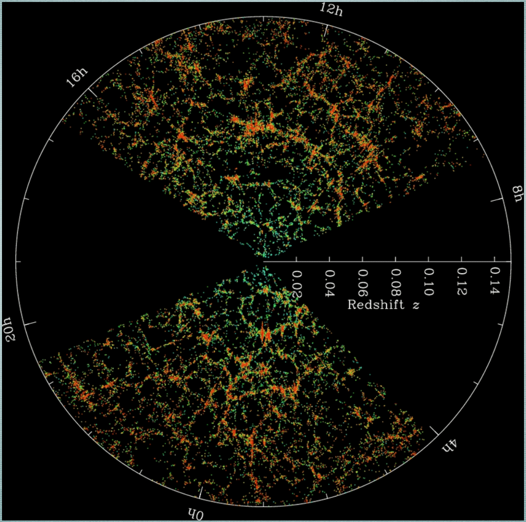


but first,

Copernician principle

Thanks to observations of the cosmic microwave background (CMB), it is known that the universe on large scales is isotropic to within $\mathcal{O}(10^{-5})$. Additionally, the distribution of matter appears to be homogeneous on scales larger than 10^8 light years.

Copernician principle



Mathematical representation

The question is: How is the geometry of a homogeneous and isotropic universe described?

Isotropy $\Longleftrightarrow$ Invariance under rotations

Homogeneity $\Longleftrightarrow$ Invariance under translations

Homo + Iso $\Longleftrightarrow$ Maximally symmetrical space

FLRW models

The Friedmann-Lemaître-Robertson-Walker (FLRW) models represent the homogeneity surfaces Σ and the fact that the universe evolves in time. In general, it can be shown that a maximally symmetric manifold, M with a metric g_{ab} has a scalar of curvature, R, constant (see for example [S. Caroll, 2004]).

FLRW models

The three metrics that exist of constant curvature are proportional to the following three metrics

1. The Euclidean spatial metric:

$$d^2\sigma = dx^2 + dy^2 + dz^2 \tag{1}$$

2. The metric of the 3-unit sphere

$$d^2\sigma = d\chi^2 + \sin^2(\theta)[d\theta^2 \sin^2(\theta) d\varphi^2] \tag{2}$$

3. The metric of the 3-unit hyperbola

$$d^2\sigma = d\chi^2 + \sinh^2(\theta)[d\theta^2 \sin^2(\theta) d\varphi^2] \tag{3}$$

FLRW models

These FLRW metrics can be expressed as

$$ds^2 = -dt^2 + a(t)^2 d\sigma^2, \tag{4}$$

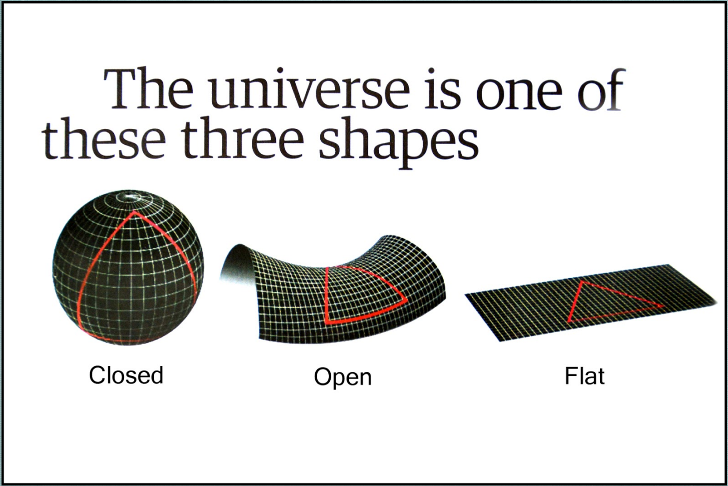
where $a(t)$ is the well known **scale factor** and codifies how big the slice Σ is at the time t. In addition, given the fact that there are no cross terms in the line element $dtdx$ and that dt^2 is not "accompanied" by spatial terms dx_i, these coordinates are given the name of **co-moving coordinates**. Observers with $dx_i = cte$ will be denoted as co-mobile or static observers.

FLRW models

Using cosmic time as a coordinate and considering the coords. spaces of Σ, we can consider the differential variety that we will use to describe space-time as $\mathbf{R} \times \Sigma$, in this way the metric takes the following form

$$dS^2 = -dt^2 + a(t)^2[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2(\theta)d\varphi^2)], \qquad (5)$$

where has it been used, $S_k(\chi) \in \{\chi, \sin(\chi), \sin h(\chi)\}$, for $k = 0, k = 1, k = -1$, respectively.


The universe is one of these three shapes
Closed
Open
Flat

FLRW models

The metric of these spacetimes can also be written as

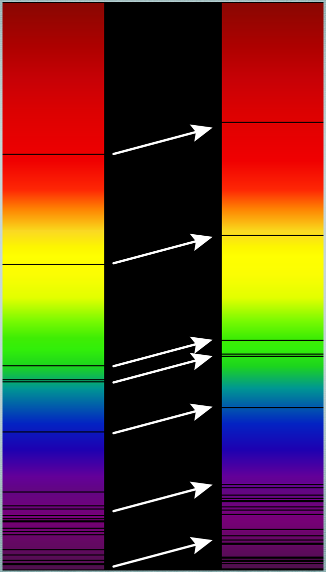
$$dS^2 = -dt^2 + a(t)^2 \left[\frac{1}{1 - kr^2} dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\varphi^2) \right], \quad (6)$$

where k can take the values $k = 0, k = 1, k = -1$.

Redshift

Next, we will define a quantity that serves as a 'measure' of accelerated expansion: how spacetime expansion alters signal sending. The way to motivate this quantity will be through an experiment with signals sent by two observers.

Redshift



Redshift

Thanks to the symmetry of space-time, generality is not lost when considering a light signal emitted at time $t = t_1$ in a radial direction by a co-mobile observer in r_1 and received by another observer with r_2. Since it is a light signal, that is, null, we can write

$$dS^2 = 0 = -dt^2 + a(t)^2 \frac{1}{1-kr^2} dr^2, \tag{7}$$

Redshift

The above implies that,

$$\int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{r_1}^{r_2} \frac{1}{\sqrt{1 - kr^2 dr}}, \tag{8}$$

If we now consider a second signal, emitted in: $t_1 + \delta_1 t$ and received in $t_2 + \delta_2 t$ it gives:

$$\int_{t_1+\delta_1 t}^{t_2+\delta_2} \frac{dt}{a(t)} = \int_{r_1+}^{r_2} \frac{1}{\sqrt{1 - kr^2 dr}}. \tag{9}$$

Redshift

Then,

$$\int_{t_1}^{t_2} \frac{1}{a(t)} dt = \int_{t_1+\delta_1 t}^{t_2 \delta_2 t} \frac{1}{a(t)} dt. \qquad (10)$$

we have the next equalities:

$$\int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{t_1}^{t_2} \frac{dt}{a(t)} + \int_{t_2}^{t_2+\delta_2} \frac{dt}{a(t)} - \int_{t_1}^{t_1+\delta_1} \frac{dt}{a(t)}. \qquad (11)$$

Redshift

The above, in the limit of short intervals, can be written as

$$\frac{\delta_1 t}{a(t_1)} = \frac{\delta_2 t}{a(t_2)} \tag{12}$$

considering δt as the time (where t is the proper time that these observers measure), between two crests of a wave and using that $\lambda = c/f$ and in this case we make $c = 1$ and $f = 1/\delta t$ we can write the above as

$$\frac{\lambda_1}{a(t_1)} = \frac{\lambda_2}{a(t_2)}, \tag{13}$$

Redshift

Note that the change in wavelength gives us direct information about the change in the scale factor. We can write the previous expression in terms of the emission frequency of a photon, ω_1 that will be observed with a lower frequency ω_2 as the universe is expanding, then we have:

$$\frac{\omega_1}{a(t_1)} = \frac{\omega_2}{a(t_2)}, \tag{14}$$

Redshift

In turn, we can rewrite as follows:

$$\frac{\omega_1}{\omega_2} = \frac{a(t_1)}{a(t_2)}, \tag{15}$$

To make the analysis easy to follow, we added the labels on ω and a:

$$\frac{\omega_{obs}}{\omega_{em}} = \frac{a_{obs}}{a_{em}}, \tag{16}$$

Redshift

This is how the “redshift” is defined between two events, which they define as the quotient of the change in wavelength

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}, \tag{17}$$

then,

$$z + 1 = \frac{a_{obs}}{a_{em}}. \tag{18}$$

Redshift

If we consider that the observations take place today, that is, $a_{obs} = a_0$, the previous expression is written as

$$z + 1 = \frac{a_0}{a_{em}}. \tag{19}$$

The **“redshift”** of an object tells us what the scale factor was when this signal was emitted.

Redshift

Now we use that the frequency ω is proportional to the energy of a photon, we find

$$z + 1 = \frac{\omega_{em}}{\omega_{obs}} = \frac{E_{em}}{E_{obs}} = \frac{a_0}{a_{em}}, \tag{20}$$

If we consider a light signal very close to the current time, t_0, it can be written

$$a_{em} = a(t_0 - \Delta t) \approx a(t_0) - \frac{da}{dt}\Delta t \tag{21}$$

Redshift

Therefore, one can also write it in terms of the frequency,

$$\frac{d\omega}{\omega} = \frac{da}{dt}\frac{\Delta t}{a_0}. \tag{22}$$

For a light signal the "distance traveled", which can be defined given a spatial hypersurface, σ and given two points.

Redshift

We can identify σ and 2 points with the following: two co-mobile observers (sending observer and receiving observer) on a hypersurface of $t = cte$, σ characterized by being the hypersurface corresponding to the moment in which the receiving observer receives the signal, said distance can be expressed as
$D = a\Delta\sigma = \Delta t$, where σ refers to the hypersurface, so

$$\frac{\delta\omega}{\omega} = \frac{da/dt}{a_0} D. \tag{23}$$

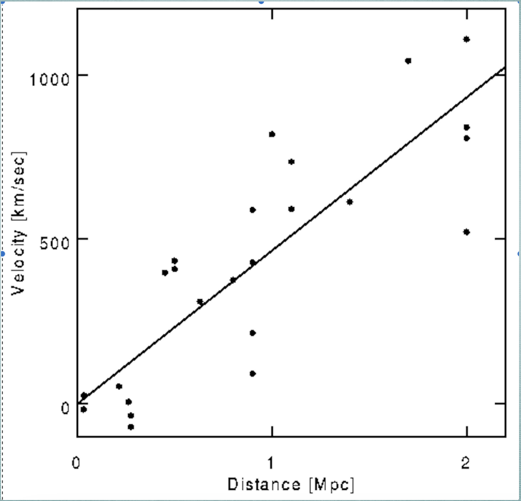
Remarks

The “redshift” of an object does not refer to the conventional Doppler effect, $-\frac{\delta\lambda}{\lambda} = \frac{\delta\omega}{\omega} = V$, in this case, it is the expansion of space and not the relative speed between observers that produces it. When distances between galaxies are observed, small compared to the **Hubble radius** H_0 and the radius of spatial curvature, the expansion of the universe looks similar to that of a group of galaxies moving in such a way that some move away from the others and the "redshift" becomes very similar to the case of the Doppler effect. For this reason, redshift is usually thought of in terms of the speed $V = z$.

Hubble law

In the above $H = \frac{\dot{a(t)}}{a(t)}$ and its value today is denoted by H_0. The aforementioned law was discovered empirically by astronomer E.Hubble in 1929, this law was the first observational result that accounts for the expansion of the universe. By means of this law he determined the value of the constant that bears his name, $H_0 = 100 km/s.MpC$. which provides a time scale, known as Hubble time T_H, today, $T_{H_0} = H_0^{-1}$ and a distance scale, called " Hubble radius" R_H today, $R_{H_0} = c \times H_0^{-1}$.

Hubble-Lemaître law



Hubble radius

Just as we can have an estimate of the Hubble radius and the Hubble time today, we can extend this concept in a “general” way for any time, this is very useful since it allows us to obtain these quantities at any t in the evolution of the universe. With $c = 1$ we can represent the time and Hubble radius as:

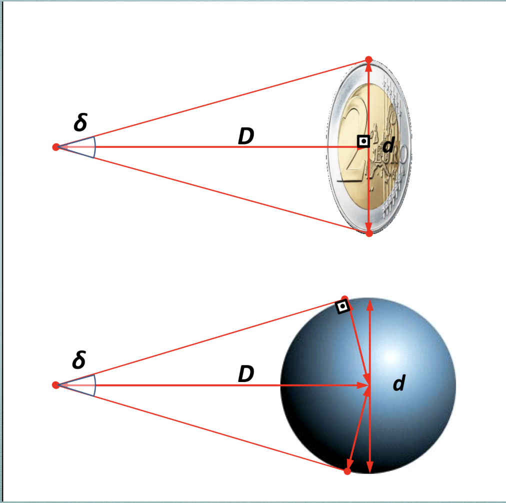
$$T_H = H(t)^{-1} \qquad R_H = H(T)^{-1}, \tag{24}$$

respectively.

Angular distance

We consider an object whose physical length is L and is oriented orthogonal to our line of sight above the intersection of the past light cone and a hypersurface at $t = T$. The value of the radial coordinate can be written as R_T when we take our world line as the origin of the spatial coordinates, let $\delta\theta$ be the angle that L subtends in the sky. We see that $L = a(T)R_T\delta\theta$

Angular distance



Angular distance

The angular distance is usually denoted by $D_A = L/\delta\theta$. On the other hand, we can consider the null radial geodesics, whose ends are in $t = T_0$ and in L, in this way we have

$$\int_T^{T_0} \frac{dt}{a(t)} = \int_0^{R_T} \frac{dr}{\sqrt{1-kr^2}} = \Psi_k(R_T). \tag{25}$$

Angular distance

Now, to determine the left member we make a change of variable, to rewrite the expression in terms of z, in this way with $z = a_0/a(t) - 1$ we have $dz = -(a_0/a(t)^2)\dot{a}(t)dt = -(a_0/a(t))H(t)dt$ and we get

$$\Psi_k(R_T) = \int_{T_T}^{T_0} \frac{dt}{a(t)} = \frac{1}{a_0}\int_0^{z_T} \frac{dz}{H(z)}, \tag{26}$$

In this way, we can compare the previous equation (25) and we obtain

$$D_A = a(T)\Psi_k^{-1}\left[(1/a_0)\int_0^{z_T} \frac{dz}{H(z)}\right]. \tag{27}$$

Particle horizon

This new definition, unlike the previous one, R_H, which only gave us an estimate of the radius of the universe at that time, allows us to have a notion of the maximum distance from which a co-mobile observer could have received light signals. , this means that we will define a distance that tells us from where causal contact could have been had.

Particle horizon

We consider the maximum distance from which a co-mobile observer at $r = 0$ and $t = t_f$ can receive light signals

$$\int_0^{t_f} \frac{1}{a(t)} dt = \int_0^{r_e} \frac{1}{\sqrt{1 - kr^2}} dr = \Psi_k(r_e). \tag{28}$$

The spatial distance at t_f will be given by

$$D = a(t_f)\Psi_k(r_e) = a(t_f) \int_0^{t_f} \frac{1}{a(t)} dt, \tag{29}$$

Particle horizon

If the integral shown below converges to the lower limit, then D represents a finite distance which will be denoted as D_{HP}

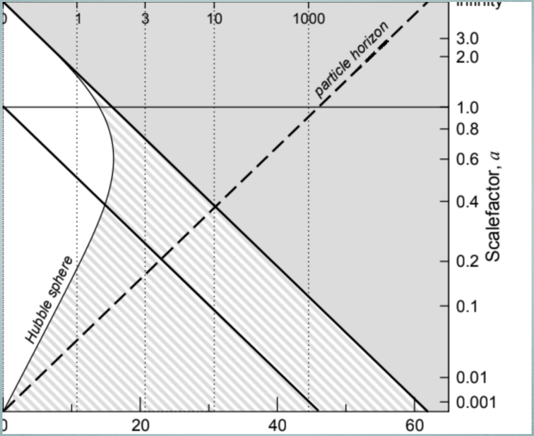
$$\int_0^{t_f} \frac{1}{a(t)} dt, \tag{30}$$

This distance is known as the particle horizon distance and will be as long as the lower limit of integration is the beginning of the universe, in this case we have $t = 0$.

Particle horizon vs Hubble radius

It is important to emphasize the similarities and differences between R_H and D_P, one way to visualize this is to calculate both quantities for the principals of the universe, which are: dust, radiation and cosmological constant.

Particle horizon vs Hubble radius



Particle horizon vs Hubble radius

Universe full of **dust**, in this case it will be found that $a(t) = At^{2/3}$, for this result we will have to

$$D_{HP}(t_f) = 3t_f, \tag{31}$$

We note that with this value for $a(t)$ we obtain that

$$H(t_f) = \frac{2}{3} t_f^{-1} \tag{32}$$

$$R_H(t_f) = H(t_f)^{-1} = \frac{3}{2} t_f \tag{33}$$

Particle horizon vs Hubble radius

Universe full of **radiation**, we will show that $a(t) = At^{1/2}$, for which the particle horizon is

$$D_{HP}(t_f) = 2t_f, \tag{34}$$

while

$$H(t_f) = \frac{1}{2} t_f^{-1} \tag{35}$$

$$R_H(t_f) = H(t_f)^{-1} = 2t_f \tag{36}$$

Particle horizon vs Hubble radius

Universe filled with **cosmological constant**, for which $a(t) = C \exp Ht$ is obtained, where $H = \sqrt{\Lambda/3}$, it is found that

$$D_{HP} = \frac{1}{H}[\exp H(t_f - t_i) - 1], \qquad (37)$$

for this case, we have

$$H(t_f) = H \qquad (38)$$

$$R_H(t_f) = \frac{1}{H} \qquad (39)$$

Dynamics

The dynamics of the universe is governed by Einstein's equations

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}, \tag{40}$$

For the FLRW metric and in this case to describe the matter of the universe, it is considered a perfect fluid,

$$T^{ab} = \rho u^a u^b + P(g^{ab} + u^a u^b), \tag{41}$$

u^a being the 4-velocity of the fluid element, ρ the density and P the pressure.

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